Ferrara–Zumino supermultiplet and the energy-momentum tensor in the lattice formulation of 4D $\mathcal{N} = 1$ SYM

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Abstract

It is well-known that Noether currents in the classical four-dimensional $\mathcal{N} = 1$ supersymmetric Yang–Mills theory (4D $\mathcal{N} = 1$ SYM), i.e., the $U(1)_A$ current, the supersymmetry (SUSY) current and the energy-momentum tensor, form a multiplet under SUSY, called the Ferrara–Zumino supermultiplet. Inspired by this structure, we define the energy-momentum tensor in the lattice formulation of 4D $\mathcal{N} = 1$ SYM by a renormalized super transformation of a lattice SUSY current. By using a renormalized SUSY Ward–Takahashi relation, the energy-momentum tensor so constructed is shown to be conserved in the quantum continuum limit. Our construction of the energy-momentum tensor is very explicit and usable in non-perturbative numerical simulations.

Keywords: Lattice gauge theory, Supersymmetry, Energy-momentum tensor

1. Introduction

Ideally, a non-perturbative formulation of a field theory with some symmetries should provide, not only the definition of correlation functions, but also the definition of renormalized Noether currents that generate correctly-normalized symmetry transformations on renormalized fields. This is expressed by renormalized Ward–Takahashi (WT) relations and, when these

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relations hold, one may say that the symmetries are *really* realized in quantum field theory.

Quite often, however, the regularization procedure breaks the preferred symmetries and, for this reason, it is generally very difficult to conclude even the existence of such renormalized Noether currents (especially when the symmetry transformations are non-linear). Even if such Noether currents are assumed to exist, the explicit construction can be very cumbersome. In the lattice formulation of supersymmetric theories, one encounters such a situation, because the infinitesimal translation (that is a part of the SUSY algebra) is broken by the spacetime lattice, implying also the breaking of SUSY. Here, almost all fundamental symmetries that define the theory are broken by the regularization.

Having the above general remark in mind, in the present paper, we study the construction of Noether currents in the lattice formulation of the four-dimensional $\mathcal{N} = 1$ supersymmetric Yang–Mills theory (4D $\mathcal{N} = 1$ SYM) [1] (see also Ref. [2] for an earlier consideration and Ref. [3] for a very readable

review). The classical Euclidean action of 4D $\mathcal{N} = 1$ SYM is given by[1]

$$S_{\text{classical}} = \int d^4x \left\{ \frac{1}{2} \operatorname{tr} \left[F_{\mu\nu}(x) F_{\mu\nu}(x) \right] + \operatorname{tr} \left[\bar{\psi}(x) \not{D} \psi(x) \right] \right\}, \qquad (1.4)$$

where the adjoint fermion (gluino) $\psi(x)$ on the Euclidean space is subject of the constraint

$$\bar{\psi}(x) = \psi^T(x)(-C^{-1}), \qquad (1.5)$$

to express the degrees of freedom of a Majorana fermion in the 4D Minkowski space. The global SUSY $\bar{\delta}_\xi$ in the classical 4D $\mathcal{N} = 1$ SYM is

$$\bar{\delta}_\xi A_\mu(x) = \bar{\xi}\gamma_\mu\psi(x),$$
$$\bar{\delta}_\xi \psi(x) = -\frac{1}{2}\sigma_{\mu\nu}\xi F_{\mu\nu}(x), \qquad \bar{\delta}_\xi\bar{\psi}(x) = \frac{1}{2}\bar{\xi}\sigma_{\mu\nu}F_{\mu\nu}(x), \qquad (1.6)$$

where the Grassmann-odd constant spinor ξ obeys $\bar{\xi} = \xi^T(-C^{-1})$. Without containing the scalar field, 4D $\mathcal{N} = 1$ SYM is the simplest supersymmetric

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[1]We basically follow the notation of Ref. [4]: The sum over repeated indices is understood. Vector indices μ, ν, ..., run over 0, 1, 2, 3. $\epsilon_{\mu\nu\rho\sigma}$ denotes the totally anti-symmetric tensor and $\epsilon_{0123} = -1$. All gamma matrices are hermitian and obey $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. We define $\gamma_5 \equiv -\gamma_0\gamma_1\gamma_2\gamma_3$ and $\sigma_{\mu\nu} \equiv [\gamma_\mu, \gamma_\nu]/2$. The charge conjugation matrix C satisfies, $C^{-1}\gamma_\mu C = -\gamma_\mu^T$, $C^{-1}\sigma_{\mu\nu}C = -\sigma_{\mu\nu}^T$, $C^{-1}\gamma_5 C = \gamma_5^T$ and $C^T = -C$. The generator of the gauge group $SU(N_c)$, T^a, is normalized as $\operatorname{tr}(T^a T^b) = (1/2)\delta^{ab}$. $F_{\mu\nu}(x)$ is the field strength $F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + ig[A_\mu(x), A_\nu(x)]$, where g is the bare gauge coupling constant, and $\not{D} \equiv \gamma_\mu D_\mu$, where $D_\mu \equiv \partial_\mu + ig[A_\mu(x),]$ is the covariant derivative in the adjoint representation. For the lattice theory, x, y, z, ... denote lattice points and a is the lattice spacing; $\hat{\mu}$ is the unit vector in the μ-direction. The link variable $U_\mu(x)$ and the gauge potential $A_\mu(x)$ are related as

$$U_\mu(x) = e^{iagA_\mu(x)}. \qquad (1.1)$$

The forward and backward difference operators respectively are defined by

$$\partial_\mu f(x) \equiv \frac{1}{a}\left[f(x + a\hat{\mu}) - f(x)\right], \qquad \partial_\mu^* f(x) \equiv \frac{1}{a}\left[f(x) - f(x - a\hat{\mu})\right], \qquad (1.2)$$

and the symmetric difference operator ∂_μ^S is defined by

$$\partial_\mu^S \equiv \frac{1}{2}(\partial_\mu + \partial_\mu^*). \qquad (1.3)$$

theory in four dimensions and we should understand the issue raised at the beginning of this paper first in this example, before tackling more complicated supersymmetric theories.

The most interesting result we will obtain below is a very explicit form of an energy-momentum tensor on the lattice $\mathcal{T}_{\mu\nu}(x)$, given by Eqs. (3.3) and (3.4), which is *conserved* in the quantum continuum limit. Our way of construction of this energy-momentum tensor was suggested by the fact that in the classical 4D $\mathcal{N} = 1$ SYM, the Noether currents associated with the $U(1)_A$ symmetry, SUSY and translational invariance, form a multiplet under SUSY, the Ferrara–Zumino (FZ) supermultiplet [5].[2] For the classical theory (1.4), we define the $U(1)_A$ current $\breve{j}_{5\mu}(x)$, the SUSY current $\breve{S}_\mu(x)$ and the energy-momentum tensor $\breve{T}_{\mu\nu}(x)$ by

$$
\begin{aligned}
\breve{j}_{5\mu}(x) &\equiv \mathrm{tr}\left[\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)\right], \\
\breve{S}_\mu(x) &\equiv -\sigma_{\rho\sigma}\gamma_\mu\,\mathrm{tr}\left[\psi(x)F_{\rho\sigma}(x)\right], \\
\breve{T}_{\mu\nu}(x) &\equiv 2\,\mathrm{tr}\left[F_{\mu\rho}(x)F_{\nu\rho}(x)\right] - \frac{1}{2}\delta_{\mu\nu}\,\mathrm{tr}\left[F_{\rho\sigma}(x)F_{\rho\sigma}(x)\right] \\
&\quad + \frac{1}{4}\,\mathrm{tr}\left[\bar{\psi}(x)\left(\gamma_\mu\overleftrightarrow{D}_\nu + \gamma_\nu\overleftrightarrow{D}_\mu\right)\psi(x)\right] - \frac{1}{2}\delta_{\mu\nu}\,\mathrm{tr}\left[\bar{\psi}(x)\overleftrightarrow{\slashed{D}}\psi(x)\right],
\end{aligned}
$$
$$(1.7)$$

where the definitions of $\breve{j}_{5\mu}(x)$ and $\breve{S}_\mu(x)$ are standard while, as well-known, the definition of the energy-momentum tensor is large extent arbitrary. In Eq. (1.7), we defined the energy-momentum tensor by first introducing a background gravitational field into the continuum action (1.4) and then taking a flat-space limit after differentiating the action with respect to the gravitational field. Then one finds, under the super transformation (1.6),

$$
\bar{\delta}_\xi \breve{j}_{5\mu}(x) = \bar{\xi}\gamma_5\breve{S}_\mu(x),
\tag{1.8}
$$

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[2]See Chapter 20 of Ref. [6] for a very readable exposition. An interesting application of the notion of the FZ multiplet in phenomenology was recently considered in Ref. [7].

and

$$\bar{\delta}_\xi \breve{S}_\mu(x) = 2\gamma_\nu \xi \left\{ \breve{T}_{\mu\nu}(x) + \frac{3}{4}\delta_{\mu\nu} \operatorname{tr}\left[\bar{\psi}(x)\slashed{D}\psi(x)\right] \right.$$

$$\left. + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\partial_\rho \breve{j}_{5\sigma}(x) + \frac{1}{4}\operatorname{tr}\left[\bar{\psi}(x)\sigma_{\mu\nu}\slashed{D}\psi(x)\right] \right\}$$

$$+ \gamma_5\gamma_\nu\xi\partial_\nu \breve{j}_{5\mu}(x)$$

$$+ \frac{3}{2}\xi \operatorname{tr}\left[\bar{\psi}(x)\gamma_\mu\slashed{D}\psi(x)\right] + \frac{3}{2}\gamma_5\xi \operatorname{tr}\left[\bar{\psi}(x)\gamma_\mu\gamma_5\slashed{D}\psi(x)\right]$$

$$- \frac{1}{2}\gamma_5\gamma_\nu\xi \operatorname{tr}\left[\bar{\psi}(x)\gamma_\mu\gamma_5\gamma_\nu\slashed{D}\psi(x)\right]$$

$$- \frac{1}{4}\sigma_{\nu\rho}\xi \operatorname{tr}\left[\bar{\psi}(x)\gamma_\mu\sigma_{\nu\rho}\slashed{D}\psi(x)\right]. \tag{1.9}$$

Similarly, $\bar{\delta}_\xi \breve{T}_{\mu\nu}(x)$ becomes a linear combination of $\partial_\rho \breve{S}_\sigma(x)$, up to terms being proportional to the equation of motion of the (massless) gluino, $\slashed{D}\psi(x)$. By integrating over the spatial coordinates of the $\mu = 0$ component of the above relations, one sees that they are consistent with the defining algebra between the SUSY charge, the $U(1)_A$ charge (this is the R-charge) and the momentum, up to the equation of motion $\slashed{D}\psi(x) = 0$. Eqs. (1.8) and (1.9) thus may be regarded as a basic characterization of fundamental Noether currents in Eq. (1.7). By going one step further, one may regard Eqs. (1.8) and (1.9) as defining relations of the Noether currents. This is the approach we adopt in the present paper. That is, we define a lattice energy-momentum tensor $\mathcal{T}_{\mu\nu}(x)$ by a (renormalized modified) super transformation of a (renormalized) lattice SUSY current $\mathcal{S}_\mu(x)$ that is conserved in the continuum limit under an appropriate gluino mass tuning [1]. Then it can be shown that the lattice energy-momentum tensor so constructed is conserved in the continuum limit. This conservation of the energy-momentum tensor is not quite trivial, because SUSY is not a manifest symmetry with the lattice regularization. For the conservation of $\mathcal{T}_{\mu\nu}(x)$, a "finiteness" of a lattice super transformation induced by $\mathcal{S}_\mu(x)$ turns to be crucial. This finiteness might be regarded as a necessary condition for the existence of a renormalized 4D $\mathcal{N} = 1$ SYM.

As the structure of our energy-momentum tensor is very explicit as Eqs. (3.3) and (3.4) show, it is usable in actual Monte Carlo simulations of 4D $\mathcal{N} = 1$ SYM [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. Physical quantities such as the viscosity (see, for exam-

ple, Ref. [33] and references therein) might be measured from the two-point function of our lattice energy-momentum tensor.

Our argument below is quite general and rather independent of the details of a lattice formulation. We assume the locality of the lattice action, that consists of the sum of a gauge boson (gluon) action S_{gluon}, the kinetic term S_{gluino} and the mass term S_{mass} of the gauge fermion (gluino),[3]

$$S \equiv S_{\text{gluon}} + S_{\text{gluino}} + S_{\text{mass}}, \tag{1.10}$$

where

$$S_{\text{gluino}} \equiv a^4 \sum_x \text{tr} \left[\bar{\psi}(x) D \psi(x) \right], \tag{1.11}$$

where D is a lattice Dirac operator $((C^{-1}D)^T = -C^{-1}D)$, and

$$S_{\text{mass}} \equiv a^4 \sum_x M \, \text{tr} \left[\bar{\psi}(x) \psi(x) \right]. \tag{1.12}$$

We further assume that the lattice action (1.10) is invariant under the hypercubic group $H(4)$, including the parity transformation $\mathcal{P}$ defined by

$$U_0(x_0, \vec{x}) \xrightarrow{\mathcal{P}} U_0(x_0, -\vec{x}), \qquad U_k(x_0, \vec{x}) \xrightarrow{\mathcal{P}} U_k^\dagger(x_0, -\vec{x} - a\hat{k}),$$
$$\psi(x_0, \vec{x}) \xrightarrow{\mathcal{P}} i\gamma_0 \psi(x_0, -\vec{x}), \qquad \bar{\psi}(x_0, \vec{x}) \xrightarrow{\mathcal{P}} -i\bar{\psi}(x_0, -\vec{x})\gamma_0, \tag{1.13}$$

where and in what follows, $\vec{x} \equiv (x_1, x_2, x_3)$ and k, $l = 1$ or 2 or 3.

In the following discussion, we make frequent use of a lattice transcription of the field strength,

$$[F_{\mu\nu}]^L (x) \equiv 2 \, \text{tr} \left[P_{\mu\nu}(x) T^a \right] T^a, \tag{1.14}$$

that is defined from the clover plaquette $P_{\mu\nu}(x)$,

$$P_{\mu\nu}(x) \equiv \frac{1}{4} \sum_{i=1}^4 \frac{1}{2ia^2g} \left[U_{i\mu\nu}(x) - U_{i\mu\nu}^\dagger(x) \right], \tag{1.15}$$

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[3]Although we use the language of a four-dimensional lattice theory in the present paper, we believe that our argument can be transcribed for a domain-wall-type [34, 35] five-dimensional setting [36, 37, 38] without much difficulty.

where

$$U_{1\mu\nu}(x) \equiv U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x),$$

$$U_{2\mu\nu}(x) \equiv U_\nu(x)U_\mu^\dagger(x-a\hat{\mu}+a\hat{\nu})U_\nu^\dagger(x-a\hat{\mu})U_\mu(x-a\hat{\mu}),$$

$$U_{3\mu\nu}(x) \equiv U_\mu^\dagger(x-a\hat{\mu})U_\nu^\dagger(x-a\hat{\mu}-a\hat{\nu})U_\mu(x-a\hat{\mu}-a\hat{\nu})U_\nu(x-a\hat{\nu}),$$

$$U_{4\mu\nu}(x) \equiv U_\nu^\dagger(x-a\hat{\nu})U_\mu(x-a\hat{\nu})U_\nu(x+a\hat{\mu}-a\hat{\nu})U_\mu^\dagger(x). \qquad (1.16)$$

Note that $[F_{\mu\nu}]^L(x)$ is traceless by construction (1.14). Under the parity, $[F_{\mu\nu}]^L(x)$ transforms in an identical manner as the continuum field strength. That is, we have

$$[F_{0k}]^L(x_0,\vec{x}) \xrightarrow{\mathcal{P}} -[F_{0k}]^L(x_0,-\vec{x}), \qquad [F_{kl}]^L(x_0,\vec{x}) \xrightarrow{\mathcal{P}} +[F_{kl}]^L(x_0,-\vec{x}).$$
$$(1.17)$$

2. Lattice SUSY current and a renormalized SUSY WT relation

In this section, we recall how the proposal of Ref. [1] that SUSY in the lattice formulation of 4D $\mathcal{N}=1$ SYM is restored with an appropriate tuning of the gluino mass M is understood in terms of the SUSY WT relation [1, 39, 20, 4]. See also Refs. [17, 19]. This discussion provides the definition of a lattice SUSY current that is conserved in the continuum limit and our basic SUSY WT relation that will be fully utilized in the construction of the energy-momentum tensor in Sec. 3.

First, as a lattice counterpart of the SUSY transformation in the continuum (1.6), we define [39]

$$\bar{\delta}_\xi U_\mu(x) \equiv iag\frac{1}{2}\bar{\xi}\gamma_\mu\left[\psi(x)U_\mu(x)+U_\mu(x)\psi(x+a\hat{\mu})\right],$$

$$\bar{\delta}_\xi U_\mu^\dagger(x) \equiv -iag\frac{1}{2}\bar{\xi}\gamma_\mu\left[U_\mu^\dagger(x)\psi(x)+\psi(x+a\hat{\mu})U_\mu^\dagger(x)\right],$$

$$\bar{\delta}_\xi\psi(x) \equiv -\frac{1}{2}\sigma_{\mu\nu}\xi\,[F_{\mu\nu}]^L(x), \qquad \bar{\delta}_\xi\bar{\psi}(x) = \frac{1}{2}\bar{\xi}\sigma_{\mu\nu}\,[F_{\mu\nu}]^L(x). \qquad (2.1)$$

To derive a WT relation, we need also the localized transformation and, corresponding to the above, we define

$$\delta_\xi U_\mu(x) \equiv iag\frac{1}{2}\left[\bar{\xi}(x)\gamma_\mu\psi(x)U_\mu(x)+\bar{\xi}(x+a\hat{\mu})\gamma_\mu U_\mu(x)\psi(x+a\hat{\mu})\right],$$

$$\delta_\xi U_\mu^\dagger(x) \equiv -iag\frac{1}{2}\left[\bar{\xi}(x)\gamma_\mu U_\mu^\dagger(x)\psi(x)+\bar{\xi}(x+a\hat{\mu})\gamma_\mu\psi(x+a\hat{\mu})U_\mu^\dagger(x)\right],$$

$$\delta_\xi\psi(x) \equiv -\frac{1}{2}\sigma_{\mu\nu}\xi(x)\,[F_{\mu\nu}]^L(x), \qquad \delta_\xi\bar{\psi}(x) = \frac{1}{2}\bar{\xi}(x)\sigma_{\mu\nu}\,[F_{\mu\nu}]^L(x), \quad (2.2)$$

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where the Grassmann-odd spinor parameter $\xi(x)$ obeys $\bar{\xi}(x) = \xi^T(x)(-C^{-1})$.

Now the lattice action (1.10) is not invariant under $\bar{\delta}_\xi$ (2.1), because, first of all, the gluino mass term (1.12) explicitly breaks SUSY and, various $O(a)$ lattice artifacts in the lattice action also break SUSY. Since the localized SUSY transformation δ_ξ (2.2) reduces to the global one (2.1) for $\xi(x) \to \xi$, the variation of the lattice action (1.10) under Eq. (2.2) can be written as

$$\delta_\xi S = a^4 \sum_x \bar{\xi}(x) \left[-\partial_\mu^S S_\mu(x) + M\chi(x) + X_S(x) \right], \tag{2.3}$$

where combinations $S_\mu(x)$ and $\chi(x)$ are defined by

$$S_\mu(x) \equiv -\sigma_{\rho\sigma}\gamma_\mu \operatorname{tr}\left\{ \psi(x) \left[F_{\rho\sigma}\right]^L(x) \right\}, \tag{2.4}$$

and

$$\chi(x) \equiv \sigma_{\mu\nu} \operatorname{tr}\left\{ \psi(x) \left[F_{\mu\nu}\right]^L(x) \right\}, \tag{2.5}$$

respectively. $S_\mu(x)$ in Eq. (2.4) is nothing but a lattice transcription of the SUSY current in the continuum, $\breve{S}_\mu(x)$ in Eq. (1.7), and $\chi(x)$ in Eq. (2.5) is an explicit SUSY breaking caused by S_{mass}. Thus, in Eq. (2.3), $X_S(x)$ represents an $O(a)$ SUSY breaking effect attributed to the lattice artifacts. Thus, considering the infinitesimal variation (2.2) in the integration variables of the functional integral, we have an exact identity that holds for any operator $\mathcal{O}$,[4]

$$\left\langle \partial_\mu^S S_\mu(x)\mathcal{O} \right\rangle = \left\langle [M\chi(x) + X_S(x)] \mathcal{O} \right\rangle - \left\langle \frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \delta_\xi \mathcal{O} \right\rangle. \tag{2.6}$$

Here is a remark on the property of $X_S(x)$ under the parity: From Eqs. (2.4) and (2.5), we see that

$$S_\mu(x_0, \vec{x}) \xrightarrow{\mathcal{P}} \begin{cases} +i\gamma_0 S_0(x_0, -\vec{x}) & \text{for } \mu = 0, \\ -i\gamma_0 S_k(x_0, -\vec{x}) & \text{for } \mu = k, \end{cases} \tag{2.7}$$

and

$$\chi(x_0, \vec{x}) \xrightarrow{\mathcal{P}} i\gamma_0 \chi(x_0, -\vec{x}). \tag{2.8}$$

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[4]Throughout this article, we always assume that an operator represented by the symbol $\mathcal{O}$ is gauge invariant.

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These shows that $\partial^S_\mu S_\mu(x)$ possesses the same transformation property as $\chi(x)$ under the parity. On the other hand, by defining $\xi(x_0, \vec{x}) \xrightarrow{\mathcal{P}} i\gamma_0 \xi(x_0, -\vec{x})$ and $\bar{\xi}(x_0, \vec{x}) \xrightarrow{\mathcal{P}} -i\bar{\xi}(x_0, -\vec{x})\gamma_0$, we see that δ_ξ in Eq. (2.2) and the parity commute to each other. Then by the parity invariance of the action S and Eqs. (2.7) and (2.8), $X_S(x)$ in Eq. (2.3) obeys

$$X_S(x_0, \vec{x}) \xrightarrow{\mathcal{P}} i\gamma_0 X_S(x_0, -\vec{x}). \tag{2.9}$$

Next, we consider the renormalization of the composite operator $X_S(x)$ (just like for the chiral symmetry [40, 41]). It can be argued that [1, 20, 4], by using the assumed symmetries of S and resultant properties of $X_S(x)$ (such as Eq. (2.9)), by writing[5]

$$
\begin{aligned}
X_S(x) = {} & (1 - \mathcal{Z}_S)\partial^S_\mu S_\mu(x) - \mathcal{Z}_T \partial^S_\mu T_\mu(x) - \frac{1}{a}\mathcal{Z}_\chi \chi(x) \\
& - \mathcal{Z}_{3F} \operatorname{tr}\left[\psi(x)\bar{\psi}(x)\psi(x)\right] \\
& - \mathcal{Z}_{\text{EOM}}\sigma_{\mu\nu}\operatorname{tr}\{[F_{\mu\nu}]^L(x)(D+M)\psi(x)\} \\
& + a\mathcal{E}(x),
\end{aligned}
\tag{2.10}
$$

and choosing renormalization constants $\mathcal{Z}_S$, $\mathcal{Z}_T$, $\mathcal{Z}_\chi$, $\mathcal{Z}_{3F}$ and $\mathcal{Z}_{\text{EOM}}$ appropriately, the dimension $11/2$ operator $\mathcal{E}(x)$ can be made at most logarithmically divergent for $a \to 0$. More precisely, the operator $\mathcal{E}(x)$ is given by a linear combination of renormalized operators with logarithmically divergent coefficients. Since the operator $\mathcal{O}$ in Eq. (2.6) is gauge invariant, the mixing of $X_S(x)$ with gauge non-invariant operators [39] does not occur in Eq. (2.10). A new lattice operator $T_\mu(x)$ in Eq. (2.10) is defined by

$$T_\mu(x) \equiv 2\gamma_\nu \operatorname{tr}\left\{\psi(x)\left[F_{\mu\nu}\right]^L(x)\right\}, \tag{2.11}$$

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[5]In Ref. [4], this operator renormalization was described by using an operator basis in the continuum theory. With the present lattice regularization, however, it is more appropriate to use an operator basis in the lattice theory as Eq. (2.10), because the renormalization is carried out in terms of counterterms with lattice structure [42, 43, 44]. Also, we adopt a different choice of basis for the term proportional to the equation of motion from Ref. [4], as this choice is practically more convenient. Finally, we make use of a simpler lattice SUSY current than Ref. [39]. Then the symmetric difference ∂^S_μ instead of the backward difference ∂^*_μ appears for the covariance under the parity.

and, from this definition,

$$T_\mu(x_0, \vec{x}) \xrightarrow{\mathcal{P}} \begin{cases} +i\gamma_0 T_0(x_0, -\vec{x}) & \text{for } \mu = 0, \\ -i\gamma_0 T_k(x_0, -\vec{x}) & \text{for } \mu = k. \end{cases} \tag{2.12}$$

From Eqs. (2.6) and (2.10), we have

$$\begin{aligned} &\left\langle \partial_\mu^S \left[\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x) \right] \mathcal{O} \right\rangle \\ &= \left(M - \frac{1}{a} \mathcal{Z}_\chi \right) \langle \chi(x) \mathcal{O} \rangle - \mathcal{Z}_{3F} \left\langle \text{tr} \left[\psi(x) \bar{\psi}(x) \psi(x) \right] \mathcal{O} \right\rangle \\ &\quad - \left\langle \frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \delta_\xi \mathcal{O} \right\rangle - \mathcal{Z}_{\text{EOM}} \left\langle \sigma_{\mu\nu} \text{tr}\{ [F_{\mu\nu}]^L(x)(D + M)\psi(x)\} \mathcal{O} \right\rangle \\ &\quad + \langle a\mathcal{E}(x)\mathcal{O} \rangle . \end{aligned} \tag{2.13}$$

In this expression, we first note

$$\left\langle \sigma_{\mu\nu} \text{tr}\{ [F_{\mu\nu}]^L(x)(D + M)\psi(x)\} \mathcal{O} \right\rangle = \left\langle \frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \delta_{F\xi} \mathcal{O} \right\rangle, \tag{2.14}$$

where the localized transformation $\delta_{F\xi}$ is a super-like transformation that acts non-trivially only on the gluino:

$$\delta_{F\xi} U_\mu(x) = 0, \qquad \delta_{F\xi} \psi(x) = \delta_\xi \psi(x), \qquad \delta_{F\xi} \bar{\psi}(x) = \delta_\xi \bar{\psi}(x), \tag{2.15}$$

because the left-hand side of Eq. (2.14) is proportional to the equation of motion of the gluino (i.e., this is the Schwinger–Dyson (SD) equation). Therefore, we have

$$\begin{aligned} &\left\langle \partial_\mu^S \left[\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x) \right] \mathcal{O} \right\rangle \\ &= \left(M - \frac{1}{a} \mathcal{Z}_\chi \right) \langle \chi(x) \mathcal{O} \rangle - \mathcal{Z}_{3F} \left\langle \text{tr} \left[\psi(x) \bar{\psi}(x) \psi(x) \right] \mathcal{O} \right\rangle \\ &\quad + \left\langle \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \Delta_\xi + a\mathcal{E}(x) \right] \mathcal{O} \right\rangle, \end{aligned} \tag{2.16}$$

where Δ_ξ is a *modified* localized super transformation,

$$\Delta_\xi \equiv \delta_\xi + \mathcal{Z}_{\text{EOM}} \delta_{F\xi}. \tag{2.17}$$

Let us consider a special case of Eq. (2.16) that $\mathcal{O}$ is a collection of renormalized local operators and the point x stays away from the support

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of $\mathcal{O}$ by a finite physical distance. In what follows, we express this situation by

$$x \leftrightsquigarrow \operatorname{supp}(\mathcal{O}). \tag{2.18}$$

Under this situation, the $\bar{\xi}(x)$-derivative in the last line of Eq. (2.16) identically vanishes.[6] Also the last term of Eq. (2.16) vanishes in the continuum limit, because for $x \leftrightsquigarrow \operatorname{supp}(\mathcal{O})$ the dimension $11/2$ operator $\mathcal{E}(x)$ does not produce any $O(1/a)$ divergence that can compensate the overall factor of a. We thus have

$$\begin{aligned}
&\left\langle \partial_\mu^S \left[\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x) \right] \mathcal{O} \right\rangle \\
&\quad \xrightarrow{a \to 0} \left(M - \frac{1}{a} \mathcal{Z}_\chi \right) \langle \chi(x)\mathcal{O} \rangle - \mathcal{Z}_{3F} \left\langle \operatorname{tr} \left[\psi(x)\bar{\psi}(x)\psi(x) \right] \mathcal{O} \right\rangle, \\
&\quad \text{for } x \leftrightsquigarrow \operatorname{supp}(\mathcal{O}).
\end{aligned} \tag{2.19}$$

This can be regarded as a would-be conservation law of a lattice SUSY current. If this relation with $\mathcal{Z}_{3F} \neq 0$ held, then the last term, that is cubic in the gluino field, would give rise to an "exotic" SUSY anomaly. This is not what we expect and we thus assume its absence:

$$\mathcal{Z}_{3F} = 0. \tag{2.20}$$

As shown in Ref. [4] by utilizing the generalized BRS transformation, this is actually the case at least to all orders in the perturbation theory. Accepting Eq. (2.20), we then have

$$\begin{aligned}
&\left\langle \partial_\mu^S \left[\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x) \right] \mathcal{O} \right\rangle \xrightarrow{a \to 0} \left(M - \frac{1}{a} \mathcal{Z}_\chi \right) \langle \chi(x)\mathcal{O} \rangle, \\
&\quad \text{for } x \leftrightsquigarrow \operatorname{supp}(\mathcal{O}).
\end{aligned} \tag{2.21}$$

The combination $M - (1/a)\mathcal{Z}_\chi$ in the right-hand side corresponds to an additive renormalization of the gluino mass. So we tune the bare mass parameter M [1] so that

$$M - \frac{1}{a} \mathcal{Z}_\chi = 0. \tag{2.22}$$

---

[6]Our argument below works when the operator $\mathcal{O}$ is ultra-local, i.e., its support is a strictly-finite region on the lattice. With the use of the overlap lattice Dirac operator D [45, 46], however, one might be interested in an operator $\mathcal{O}$ that is not ultra-local but exponentially local [47]. Although we do not give any analysis for such a case, on physical grounds, we believe that our conclusions will not change.

In actual numerical simulations, this tuning should be carried out for each value of the lattice spacing a. In this way, we obtain the conservation law in the continuum limit:

$$\left\langle \partial_\mu^S \left[\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x) \right] \mathcal{O} \right\rangle \xrightarrow{a \to 0} 0, \qquad \text{for } x \leftrightsquigarrow \mathrm{supp}(\mathcal{O}), \qquad (2.23)$$

that is a minimal requirement for the restoration of SUSY in the continuum limit.

We consider the renormalization of the conserved current $\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x)$. Since this is a dimension 7/2 gauge invariant vectorial operator, it can mix only with the operators $S_\mu(x)$ and $T_\mu(x)$. However, if the mixing were generic, it would be inconsistent with the conservation law (2.23). That is, the renormalization must be multiplicative.[7] Thus we set

$$\mathcal{S}_\mu(x) \equiv \mathcal{Z} \left[\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x) \right], \qquad (2.24)$$

and call this a renormalized lattice SUSY current. With an appropriate choice of the constant $\mathcal{Z}$, $\mathcal{S}_\mu(x)$ should have finite correlation functions with any renormalized operator $\mathcal{O}$, as far as $x \leftrightsquigarrow \mathrm{supp}(\mathcal{O})$. The renormalization constants $\mathcal{Z}_S$ and $\mathcal{Z}_T$ in Eqs. (2.10) and (2.24) emerge from the power-divergence subtraction and it can be argued that such constants are independent of the renormalization scale μ^2 and finite as $a \to 0$ [41]. On the other hand, by the dimensional reason the constant $\mathcal{Z}$ in Eq. (2.24) is at most logarithmically divergent. In terms of the renormalized SUSY current (2.24), the SUSY WT relation (2.16) under the assumptions (2.20) and (2.22) reads

$$\left\langle \partial_\mu^S \mathcal{S}_\mu(x) \mathcal{O} \right\rangle = \left\langle \mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \Delta_\xi + a\mathcal{E}(x) \right] \mathcal{O} \right\rangle. \qquad (2.25)$$

Finally, we argue that for any renormalized local operator $\mathcal{O}$, the combination

$$\mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \Delta_\xi + a\mathcal{E}(x) \right] \mathcal{O}, \qquad (2.26)$$

when $x \in \mathrm{supp}(\mathcal{O})$, produces a renormalized local operator in the support of $\mathcal{O}$. In Eq. (2.26), something non-trivial in the continuum limit can occur only when $x \in \mathrm{supp}(\mathcal{O})$, because otherwise the $\bar{\xi}(x)$-derivative vanishes and

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[7]It is very interesting to confirm this renormalizability by the lattice perturbation theory along the line of Ref. [39].

the dimension 11/2 operator $\mathcal{E}(x)$ does not produce any $O(1/a)$ divergence that can compensate the overall factor of a. To see what happens when $x \in \mathrm{supp}(\mathcal{O})$, we sum Eq. (2.25) over the point x within a finite region $\mathcal{D}_\mathcal{O}$ that contains the operator $\mathcal{O}$. Then, since $\sum_{x \in \mathcal{D}_\mathcal{O}} \partial_\mu^S \mathcal{S}_\mu(x)$ in the left-hand side becomes a collection of (mutually non-overlapping) renormalized operators that have no overlap with $\mathcal{O}$, the left-hand side remains finite as $a \to 0$. This shows that the sum $\sum_{x \in \mathcal{D}_\mathcal{O}}$ of the right-hand side of Eq. (2.25) is finite too and we see that the combination (2.26) does not produce any ultraviolet divergence even for $x \in \mathrm{supp}(\mathcal{O})$. We note that such a finiteness will be indispensable for the existence of a renormalized SYM, because for that there should exist a renormalized SUSY current that generates a finite super transformation on renormalized fields. Since the above analysis shows that $\mathcal{S}_\mu(x)$ is the unique conserved SUSY current in the present framework, $\mathcal{S}_\mu(x)$ should generate such a finite super transformation. This argument on the basis of the existence of a renormalized SYM also supports the finiteness of the combination (2.26).

3. A lattice energy-momentum tensor and its conservation law

We now define an energy-momentum tensor $\mathcal{T}_{\mu\nu}(x)$ on the lattice through the relation quite analogous to Eq. (1.9). That is,

$$
\begin{aligned}
\mathcal{Z}\bar{\Delta}_\xi \mathcal{S}_\mu(x) \equiv 2\gamma_\nu \xi \big\{ &\mathcal{T}_{\mu\nu}(x) + c\delta_{\mu\nu}\,\mathrm{tr}\left[\bar{\psi}(x)(D+M)\psi(x)\right] \\
&+ (\text{terms anti-symmetric in } \mu \text{ and } \nu) \big\} \\
&+ (\text{terms proportional to } \gamma_5\gamma_\nu\xi,\ \xi,\ \gamma_5\xi,\ \sigma_{\nu\rho}\xi), \quad (3.1)
\end{aligned}
$$

where c is a constant and $\bar{\Delta}_\xi$ is a "global version" of the localized super transformation (2.17), i.e.,

$$
\bar{\Delta}_\xi \equiv \bar{\delta}_\xi + \mathcal{Z}_{\mathrm{EOM}}\bar{\delta}_{F\xi}, \quad (3.2)
$$

and $\bar{\delta}_{F\xi}$ is obtained by setting $\xi(x) \to \xi$ in Eq. (2.15). One can invert Eq. (3.1) with respect to $\mathcal{T}_{\mu\nu}(x)$: We introduce (here and in what follows, indices α, β, ..., which run over 1, 2, 3, 4, denote the spinor indices),

$$
\begin{aligned}
\Theta_{\mu\nu}(x) &\equiv \frac{1}{8}(\gamma_\nu)_{\beta\alpha}\frac{\partial}{\partial\xi_\beta}\left[\mathcal{Z}\bar{\Delta}_\xi \mathcal{S}_\mu(x)\right]_\alpha \\
&= \mathcal{Z}^2 \mathcal{Z}_S \frac{1}{8}(\gamma_\nu)_{\beta\alpha}\frac{\partial}{\partial\xi_\beta}\left\{\left(\bar{\delta}_\xi + \mathcal{Z}_{\mathrm{EOM}}\bar{\delta}_{F\xi}\right)\left[S_\mu(x) + \frac{\mathcal{Z}_T}{\mathcal{Z}_S}T_\mu(x)\right]\right\}_\alpha,
\end{aligned}
$$
$$
(3.3)
$$

where in the second equality we have substituted Eqs. (3.2) and (2.24), then

$$\mathcal{T}_{\mu\nu}(x) = \frac{1}{2} \left[\Theta_{\mu\nu}(x) + \Theta_{\nu\mu}(x) \right] - c\delta_{\mu\nu} \operatorname{tr} \left[\bar{\psi}(x)(D + M)\psi(x) \right]. \qquad (3.4)$$

From relations obtained so far, one can confirm that $\Theta_{\mu\nu}(x)$ in Eq. (3.3) behaves as a second-rank tensor under the parity:[8]

$$\Theta_{\mu\nu}(x_0, \vec{x}) \xrightarrow{\mathcal{P}} \begin{cases} +\Theta_{00}(x_0, -\vec{x}) & \text{for } \mu = \nu = 0, \\ -\Theta_{k0}(x_0, -\vec{x}) & \text{for } \mu = k \text{ and } \nu = 0, \\ -\Theta_{0k}(x_0, -\vec{x}) & \text{for } \mu = 0 \text{ and } \nu = k, \\ +\Theta_{kl}(x_0, -\vec{x}) & \text{for } \mu = k \text{ and } \nu = l. \end{cases} \qquad (3.5)$$

The defining relation (3.1) is quite analogous to the classical relation (1.9). Yet they differ in that the super transformation $\bar{\delta}_\xi$ is replaced by the renormalized modified super transformation $\mathcal{Z}\bar{\Delta}_\xi$ and the Dirac operator is shifted by the gluino mass M; also the coefficient of the term that is proportional to the fermion action is changed from $3/4$ to c.

Quite interestingly, we can show that the energy-momentum tensor defined by Eq. (3.4) is conserved in the quantum continuum limit. That is

$$\left\langle \partial_\mu^S \mathcal{T}_{\mu\nu}(x)\mathcal{O} \right\rangle \xrightarrow{a \to 0} 0, \qquad \text{for } x \longleftrightarrow \operatorname{supp}(\mathcal{O}), \qquad (3.6)$$

for any renormalized local operator $\mathcal{O}$. Note that this conservation law holds for any value of c in Eq. (3.4) because the term $\operatorname{tr}[\bar{\psi}(x)(D + M)\psi(x)]$ is proportional to the equation of motion and it has no correlation with $\mathcal{O}$ when x is not in the support of $\mathcal{O}$. When $x \notin \operatorname{supp}(\mathcal{O})$, one can replace it by

$$\left\langle \operatorname{tr} \left[\bar{\psi}(x)(D + M)\psi(x) \right] \right\rangle = -2(N_c^2 - 1)a^{-4}. \qquad (3.7)$$

Since this is independent of x, the last term in Eq. (3.4) does not affect the conservation law. The most general structure of the energy-momentum tensor (that is symmetric in its indices) in the lattice formulation of the Yang–Mills theory coupled to fermions has been given in Ref. [48]; see also Refs. [49, 50, 51] for related analyses. According to Ref. [48], a symmetric

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[8]To see this, it is convenient to note that $\bar{\Delta}_\xi$ and the parity commute to each other, if one assigns $\xi \xrightarrow{\mathcal{P}} i\gamma_0\xi$ and $\bar{\xi} \xrightarrow{\mathcal{P}} -i\bar{\xi}\gamma_0$.

energy-momentum tensor that satisfies the conservation law in the continuum limit (3.6), if it exists, is essentially unique. Only ambiguities are the overall normalization ($\mathcal{Z}^2\mathcal{Z}_S$ in Eq. (3.3) in our construction) and a proportionality constant to the fermion Lagrangian (i.e., the constant c in Eq. (3.4)). The bottom line is that our definition (3.4), that is suggested from the structure of the FZ supermultiplet, gives rise to a very explicit form of this unique conserved symmetric energy-momentum tensor.

To show the conservation law (3.6), we first show the conservation law of $\Theta_{\mu\nu}(x)$ in Eq. (3.3) that is not necessary symmetric under $\mu \leftrightarrow \nu$. Then we show the anti-symmetric part of $\Theta_{\mu\nu}(x)$ also is conserved in the continuum limit, implying Eq. (3.6).

To show the conservation law of $\Theta_{\mu\nu}(x)$, we set $\mathcal{O} \to \partial_\nu^S \mathcal{S}_\nu(y)\mathcal{O}$ in the WT relation (2.25). This yields

$$
\begin{aligned}
&\left\langle \left[\partial_\mu^S \mathcal{S}_\mu(x)\right]_\alpha \left[\partial_\nu^S \mathcal{S}_\nu(y)\right]_\beta \mathcal{O}\right\rangle \\
&= \left\langle \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x)\right]_\alpha \left[\partial_\nu^S \mathcal{S}_\nu(y)\right]_\beta \mathcal{O}\right\rangle \\
&= -\left\langle \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(y)}\Delta_\xi + a\mathcal{E}(y)\right]_\beta \left[\partial_\mu^S \mathcal{S}_\mu(x)\right]_\alpha \mathcal{O}\right\rangle \\
&= \left\langle \frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}_\beta(y)} \left[\mathcal{Z}\Delta_\xi\partial_\mu^S \mathcal{S}_\mu(x)\right]_\alpha \mathcal{O}\right\rangle \\
&\quad + \left\langle \left[\partial_\mu^S \mathcal{S}_\mu(x)\right]_\alpha \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(y)}\Delta_\xi + a\mathcal{E}(y)\right]_\beta \mathcal{O}\right\rangle. \qquad (3.8)
\end{aligned}
$$

The second equality holds because the first expression is anti-symmetric under the exchange, $x \leftrightarrow y$, $\mu \leftrightarrow \nu$ and $\alpha \leftrightarrow \beta$. Using Eq. (2.25) once again in the last line, we have the identity

$$
\begin{aligned}
&\left\langle \frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}_\beta(y)} \left[\mathcal{Z}\Delta_\xi\partial_\mu^S \mathcal{S}_\mu(x)\right]_\alpha \mathcal{O}\right\rangle \\
&= \left\langle \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x)\right]_\alpha \left[\partial_\nu^S \mathcal{S}_\nu(y)\right]_\beta \mathcal{O}\right\rangle \\
&\quad - \left\langle \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x)\right]_\alpha \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(y)}\Delta_\xi + a\mathcal{E}(y)\right]_\beta \mathcal{O}\right\rangle.
\end{aligned}
$$
$$(3.9)$$

We then sum this relation over y within a finite region $\mathcal{D}_x$ containing the operator $\partial_\mu^S \mathcal{S}_\mu(x)$. Noting the identity between the local and global transformations,

$$\sum_{y \in \mathcal{D}_x} \frac{\partial}{\partial \bar{\xi}_\beta(y)} \Delta_\xi \partial_\mu^S \mathcal{S}_\mu(x) = \frac{\partial}{\partial \bar{\xi}_\beta} \bar{\Delta}_\xi \partial_\mu^S \mathcal{S}_\mu(x), \tag{3.10}$$

we have

$$\left\langle \frac{\partial}{\partial \bar{\xi}_\beta} \left[\mathcal{Z} \bar{\Delta}_\xi \partial_\mu^S \mathcal{S}_\mu(x) \right]_\alpha \mathcal{O} \right\rangle$$

$$= \left\langle \mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \Delta_\xi + a\mathcal{E}(x) \right]_\alpha a^4 \sum_{y \in \mathcal{D}_x} \left[\partial_\nu^S \mathcal{S}_\nu(y) \right]_\beta \mathcal{O} \right\rangle$$

$$- \left\langle \mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \Delta_\xi + a\mathcal{E}(x) \right]_\alpha a^4 \sum_{y \in \mathcal{D}_x} \mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(y)} \Delta_\xi + a\mathcal{E}(y) \right]_\beta \mathcal{O} \right\rangle. \tag{3.11}$$

Since $\bar{\xi} = \xi^T(-C^{-1})$ implies

$$(\gamma_\nu)_{\beta\alpha} \frac{\partial}{\partial \xi_\beta} \left[\mathcal{Z} \bar{\Delta}_\xi \partial_\mu^S \mathcal{S}_\mu(x) \right]_\alpha = (C^{-1}\gamma_\nu)_{\alpha\beta} \frac{\partial}{\partial \bar{\xi}_\beta} \left[\mathcal{Z} \bar{\Delta}_\xi \partial_\mu^S \mathcal{S}_\mu(x) \right]_\alpha, \tag{3.12}$$

Eqs. (3.3) and (3.11) yield,

$$\langle \partial_\mu^S \Theta_{\mu\nu}(x) \mathcal{O} \rangle$$

$$= \frac{1}{8}(C^{-1}\gamma_\nu)_{\alpha\beta} \left\langle \mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \Delta_\xi + a\mathcal{E}(x) \right]_\alpha a^4 \sum_{y \in \mathcal{D}_x} \left[\partial_\nu^S \mathcal{S}_\nu(y) \right]_\beta \mathcal{O} \right\rangle$$

$$- \frac{1}{8}(C^{-1}\gamma_\nu)_{\alpha\beta} \left\langle \mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \Delta_\xi + a\mathcal{E}(x) \right]_\alpha \right.$$

$$\left. \times a^4 \sum_{y \in \mathcal{D}_x} \mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(y)} \Delta_\xi + a\mathcal{E}(y) \right]_\beta \mathcal{O} \right\rangle. \tag{3.13}$$

Now, let us suppose that the point x stays away from the support of the renormalized operator $\mathcal{O}$ by a finite physical distance, i.e., $x \longleftrightarrow \text{supp}(\mathcal{O})$. Let us further assume that the region $\mathcal{D}_x$ is chosen so that it does not overlap

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with the support of $\mathcal{O}$. In this situation, since $\sum_{y \in \mathcal{D}_x} \partial_\nu^S \mathcal{S}_\nu(y)$ does not have any support at x, Eq. (3.13) reduces to

$$
\begin{aligned}
&\left\langle \partial_\mu^S \Theta_{\mu\nu}(x) \mathcal{O} \right\rangle \\
&= \frac{1}{8} (C^{-1} \gamma_\nu)_{\alpha\beta} \left\langle \mathcal{Z} \left[a\mathcal{E}(x) \right]_\alpha a^4 \sum_{y \in \mathcal{D}_x} \left[\partial_\nu^S \mathcal{S}_\nu(y) \right]_\beta \mathcal{O} \right\rangle \\
&\quad - \frac{1}{8} (C^{-1} \gamma_\nu)_{\alpha\beta} \left\langle a^4 \sum_{y \in \mathcal{D}_x} \mathcal{Z} \left[-\frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \Delta_\xi + a\mathcal{E}(x) \right]_\alpha \mathcal{Z} \left[a\mathcal{E}(y) \right]_\beta \mathcal{O} \right\rangle .
\end{aligned}
$$
(3.14)

The first term in the right-hand side is a correlation function of renormalized operators without any mutual overlap with the overall factor of a. Thus, this term vanishes in the $a \to 0$ limit. In the second term in the right-hand side, we use the property argued around Eq. (2.26). Then it is also a correlation function of renormalized operators without any mutual overlap with the overall factor of a. Thus this also vanishes in the $a \to 0$ limit. In this way, we conclude

$$
\left\langle \partial_\mu^S \Theta_{\mu\nu}(x) \mathcal{O} \right\rangle \xrightarrow{a \to 0} 0, \qquad \text{for } x \longleftrightarrow \text{supp}(\mathcal{O}),
$$
(3.15)

for any renormalized operator $\mathcal{O}$.

Next, we consider the anti-symmetric part of $\Theta_{\mu\nu}(x)$,

$$
\mathcal{A}_{\mu\nu}(x) \equiv \frac{1}{2} \left[\Theta_{\mu\nu}(x) - \Theta_{\nu\mu}(x) \right],
$$
(3.16)

and its renormalization. It must be possible to expand this composite operator by gauge invariant operators with dimensions less than or equal to 4 that behave in the same way under the hypercubic group as the right-hand side of Eq. (3.16) (for the parity, see Eq. (3.5)). By taking also the constraint (1.5) into account, the most general possibility turns to be[9]

$$
\begin{aligned}
&\mathcal{A}_{\mu\nu}(x) \\
&= A_1 \epsilon_{\mu\nu\rho\sigma} \partial_\rho^S \, \text{tr} \left[\bar{\psi}(x) \gamma_\sigma \gamma_5 \psi(x) \right] + A_2 \, \text{tr} \left[\bar{\psi}(x) \sigma_{\mu\nu} (D + M) \psi(x) \right] + a\mathcal{G}_{\mu\nu}(x).
\end{aligned}
$$
(3.17)

---

[9] For example, a seemingly-obvious candidate $\text{tr}[\bar{\psi}(x)\sigma_{\mu\nu}\psi(x)]$ identically vanishes because of Eq. (1.5); another candidate $\text{tr}[\bar{\psi}(x)\gamma_\mu D_\nu^S \psi(x)] - (\mu \leftrightarrow \nu)$, where $D_\mu^S f(x) \equiv (1/2a)[U_\mu(x)f(x+a\hat{\mu})U_\mu^\dagger(x) - U_\mu(x-a\hat{\mu})^\dagger f(x-a\hat{\mu})U_\mu(x-a\hat{\mu})]$, reduces to Eq. (3.17), by the fact that $D = \slashed{D} + O(a)$ and $\sigma_{\mu\nu}\slashed{D} = -\epsilon_{\mu\nu\rho\sigma}D_\rho\gamma_\sigma\gamma_5 + \gamma_\mu D_\nu - \gamma_\nu D_\mu$.

That is, by choosing the constants A_1 and A_2 appropriately, the dimension 5 operator $\mathcal{G}_{\mu\nu}(x)$ can be made at most logarithmically divergent. Now, quite fortunately, Eq. (3.17) shows that the anti-symmetric part is conserved by itself:

$$\langle \partial_\mu^S \mathcal{A}_{\mu\nu}(x)\mathcal{O}\rangle \xrightarrow{a\to 0} 0, \qquad \text{for } x \nleftrightarrow \text{supp}(\mathcal{O}). \qquad (3.18)$$

This is trivially true for the first term of Eq. (3.17). For the second term, this follows from the SD equation (the equation of motion of $\psi(x)$). Finally, the last term of Eq. (3.17) vanishes in the continuum limit. Combined Eqs. (3.18) and (3.15), we conclude that the symmetric part of $\Theta_{\mu\nu}(x)$, $(1/2)[\Theta_{\mu\nu}(x) + \Theta_{\nu\mu}(x)]$, also is conserved. This proves our assertion Eq. (3.6), because the symmetric part of $\Theta_{\mu\nu}(x)$ is the first term of Eq. (3.4) and the term with the coefficient c does not affect the conservation law as we already explained.

We note that, with our definition of the energy-momentum tensor (3.4), the trace anomaly [52, 53, 54, 55] and the gamma-trace anomaly (superconformal anomaly) [56, 57, 58, 59, 60, 61, 62, 63, 64] are related by[10]

$$\langle \mathcal{T}_{\mu\mu}(x)\mathcal{O}\rangle = \frac{1}{8}\left\langle \frac{\partial}{\partial\xi_\alpha}\left[\mathcal{Z}\bar{\Delta}_\xi\gamma_\mu\mathcal{S}_\mu(x)\right]_\alpha\mathcal{O}\right\rangle + 8c(N_c^2 - 1)a^{-4}\langle\mathcal{O}\rangle, \qquad (3.19)$$

where we have used Eq. (3.7) assuming $x \notin \text{supp}(\mathcal{O})$.

Now we discuss how our energy-momentum tensor (3.4) can be used in actual non-perturbative Monte Carlo simulations. As Eqs. (3.3) and (3.4) show, the definition of the energy-momentum tensor contains four combinations of renormalization constants, $\mathcal{Z}^2\mathcal{Z}_S$, $\mathcal{Z}_{\text{EOM}}$, $\mathcal{Z}_T/\mathcal{Z}_S$ and c; we have to know these numbers to construct the energy-momentum tensor. Among these, the ratio $\mathcal{Z}_T/\mathcal{Z}_S$ has been measured [66, 16, 18, 22, 25] by determining the coefficients in the relation ($\mathcal{O}$ is a fermionic spinorial operator)

$$\partial_\mu^S\langle S_\mu(x)\mathcal{O}\rangle + \frac{\mathcal{Z}_T}{\mathcal{Z}_S}\partial_\mu^S\langle T_\mu(x)\mathcal{O}\rangle = \frac{1}{\mathcal{Z}_S}\left(M - \frac{1}{a}\mathcal{Z}_\chi\right)\langle\chi(x)\mathcal{O}\rangle. \qquad (3.20)$$

This is Eq. (2.21) up to $O(a)$ corrections. Thus we already know that the number $\mathcal{Z}_T/\mathcal{Z}_S$ can be determined numerically (up to $O(a)$ corrections). Next, to determine $\mathcal{Z}_{\text{EOM}}$ in Eq. (3.3), we may use the conservation law of $\Theta_{\mu\nu}(x)$, Eq. (3.15), itself.

The determinations of $\mathcal{Z}^2\mathcal{Z}_S$ and c are somewhat correlated. We first note that, when the energy-momentum tensor $\mathcal{T}_{\mu\nu}(x)$ (3.4) is inserted in a

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[10]Ref. [65] is a pioneering work on the trace anomaly in lattice gauge theory.

physical amplitude, the last term of Eq. (3.4) just gives rise to an additive constant (3.7) that is identical for *any* physical amplitudes; the constant c thus corresponds to a choice of the origin of the energy. If we consider the difference in expectation values of $\mathcal{T}_{\mu\nu}(x)$ in two physical states, therefore, the contribution of the last term of Eq. (3.4) cancels out and it is the same as the difference in expectation values of $(1/2)[\Theta_{\mu\nu}(x) + \Theta_{\nu\mu}(x)]$. Noting this fact, one may determine $\mathcal{Z}^2\mathcal{Z}_S$, the absolute normalization of $\Theta_{\mu\nu}(x)$ in Eq. (3.3), by setting the difference of expectation values of the "energy operator" $-a^3\sum_{\vec{x}}\Theta_{00}(x)$ in two different physical states to a certain prescribed value.

Although the value of c influences neither on the conservation law nor on the difference in expectation values of the energy-momentum tensor, there exists a physically natural choice of c in the present supersymmetric system. That is, we may require that the expectation value of the energy density to vanish

$$\langle\mathcal{T}_{00}(x)\rangle = 0, \tag{3.21}$$

when *periodic boundary conditions* are imposed on all the fields. This requirement fixes

$$c = -\frac{a^4}{2(N_c^2 - 1)}\langle\Theta_{00}(x)\rangle_{\text{periodic boundary conditions}}, \tag{3.22}$$

because of Eq. (3.7). Eq. (3.21) states that the derivative of the supersymmetric partition function (i.e., the Witten index [67]) with respect to the temporal size of the system vanishes; this holds only if a choice of the origin of the energy is consistent with the SUSY algebra. In other words, Eq. (3.21) is required for the spatial integral $-a^3\sum_{\vec{x}}\mathcal{T}_{00}(x)$ to be the energy operator appearing in the right-hand side of the SUSY algebra. It is thus a natural requirement from the perspective of SUSY.[11] This completes our

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[11]We emphasize that Eq. (3.21) should hold even in a theory in which SUSY is spontaneously broken (recall that the Witten index is independent of the temporal size of the system even in such a case). This same idea was adopted to find an expression of the energy density in a lattice formulation of the two-dimensional (2D) $\mathcal{N} = (2, 2)$ SYM [68, 69, 70]. For this 2D system, fortunately, there exist lattice formulations that possess one exact fermionic symmetry [71, 72, 73, 74] and, employing this symmetry, one can define the energy density operator that shows the zero-point energy consistent with SUSY. This energy density operator on the lattice has been used [75] to measure the vacuum energy density associated with a possible dynamical SUSY breaking in the above 2D system.

non-perturbative construction of a symmetric energy-momentum tensor. It is conserved in the quantum continuum limit and, by choosing c as Eq. (3.22), the origin of the energy is consistent with SUSY.

A remaining important issue is whether our symmetric energy-momentum tensor (3.4) generates, through a renormalized WT relation, a correctly-normalized translation on renormalized fields. Although we do not go into this question in the present paper, assuming the existence of a renormalized translational invariant theory and considering the uniqueness of the conserved symmetric energy-momentum tensor in the continuum limit [48], we believe that the answer is affirmative: Of course, further consideration is required on this point.

4. Conclusion

We have presented a non-perturbative construction of a symmetric energy-momentum tensor in the lattice formulation of 4D $\mathcal{N} = 1$ SYM. Inspired by the relation (1.9) in the FZ supermultiplet, we defined the energy-momentum tensor by a renormalized modified super transformation of a renormalized SUSY current that generates a finite super transformation on renormalized operators (under the tuning of the gluino mass). Then, it can be shown that the lattice energy-momentum tensor is conserved in the quantum continuum limit. The resulting energy-momentum tensor may be used to measure physical quantities related with the energy-momentum tensor (such as the viscosity) by numerical simulations.

A question naturally arises is then what will be resulted from the consideration on the another relation in the FZ multiplet, Eq. (1.8), a relation between the $U(1)_A$ current and the SUSY current. It is clear that the classical relation (1.8) as it stands cannot hold in quantum theory, because the $U(1)_A$ current is not conserved owing to the axial anomaly, while we are believing that SUSY does not suffer from any anomaly. Ironically, it turns out that the situation for this seemingly simple relation (1.8) is much more complicated than for (seemingly complicated) Eq. (1.9); this is precisely because of the axial anomaly. We hope to come back this problem in the near future. If the full structure of the FZ multiplet (with corrections by the anomaly) can be realized in a well-regularized framework such as the lattice, it will be a quite useful starting point to understand the so-called anomaly puzzle [76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86].

Note added in proof

Eq. (3.13) implies the following very suggestive relation: Noting that $\sum_{y \in \mathcal{D}_x} \partial_\nu^S \mathcal{S}_\nu(y)$ in the first term of the right-hand side of Eq. (3.13) does not have the support at $y = x$, using the SUSY WT relation (2.25) again, we have

$$a^4 \sum_{x \in \mathcal{D}_{\mathcal{O}}} \langle \partial_\mu^S \Theta_{\mu\nu}(x)\mathcal{O} \rangle$$

$$= -\frac{1}{8}(C^{-1}\gamma_\nu)_{\alpha\beta}\, a^4 \sum_{x \in \mathcal{D}_{\mathcal{O}}} a^4 \sum_{y \in \mathcal{D}_x}$$

$$\times \left\langle \left\{ \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x)\right]_\alpha, \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(y)}\Delta_\xi + a\mathcal{E}(y)\right]_\beta \right\}\mathcal{O} \right\rangle, \tag{4.1}$$

where $\mathcal{D}_{\mathcal{O}}$ is a finite region that contains the operator $\mathcal{O}$ entirely. On the other hand, for the anti-symmetric part of $\Theta_{\mu\nu}(x)$, $\mathcal{A}_{\mu\nu}(x)$ in Eq. (3.16),

$$a^4 \sum_{x \in \mathcal{D}_{\mathcal{O}}} \langle \partial_\mu^S \mathcal{A}_{\mu\nu}(x)\mathcal{O} \rangle \xrightarrow{a \to 0} 0, \tag{4.2}$$

because Eq. (3.17) shows that $\langle \mathcal{A}_{\mu\nu}(x)\mathcal{O} \rangle$ is proportional to the delta function $\delta^4(x-z)$ in the continuum limit, where z is any point in the support of $\mathcal{O}$. Similarly, for the last term of Eq. (3.4), we have

$$a^4 \sum_{x \in \mathcal{D}_{\mathcal{O}}} \langle \partial_\mu^S \delta_{\mu\nu}\, \mathrm{tr}\left[\bar{\psi}(x)(D+M)\psi(x)\right]\mathcal{O} \rangle = 0. \tag{4.3}$$

Thus, combining the above three relations,

$$a^4 \sum_{x \in \mathcal{D}_{\mathcal{O}}} \langle \partial_\mu^S \mathcal{T}_{\mu\nu}(x)\mathcal{O} \rangle$$

$$\xrightarrow{a \to 0} -\frac{1}{8}(C^{-1}\gamma_\nu)_{\alpha\beta}\, a^4 \sum_{x \in \mathcal{D}_{\mathcal{O}}} a^4 \sum_{y \in \mathcal{D}_x}$$

$$\times \left\langle \left\{ \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x)\right]_\alpha, \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(y)}\Delta_\xi + a\mathcal{E}(y)\right]_\beta \right\}\mathcal{O} \right\rangle. \tag{4.4}$$

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This may be regarded as the SUSY algebra in the present lattice framework if we assume that the lattice energy-momentum tensor $\mathcal{T}_{\mu\nu}$ generates the translation (on gauge invariant operators $\mathcal{O}$) as

$$a^4 \sum_{x \in \mathcal{D}_\mathcal{O}} \left\langle \partial_\mu^S \mathcal{T}_{\mu\nu}(x) \mathcal{O} \right\rangle \xrightarrow{a \to 0} - \left\langle \partial_\nu \mathcal{O} \right\rangle . \qquad (4.5)$$

Remark in proof

A quite similar but somewhat different definition of a lattice energy-momentum tensor in 4D $\mathcal{N} = 1$ SYM, which is superior in several aspects compared with the one in the present note, has been given in Ref. [87].

Acknowledgements

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